MagLev Lab

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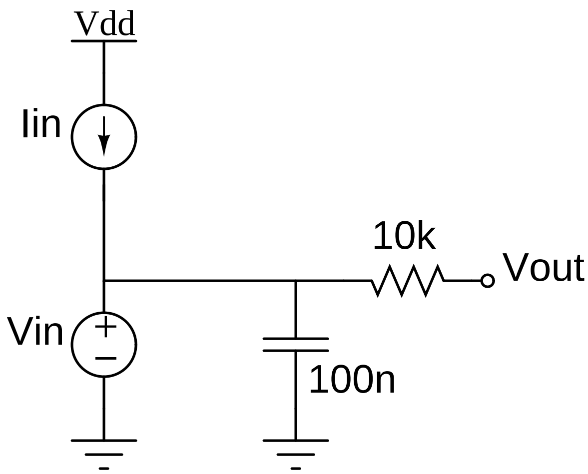
The birth of a control system

As I start to type up this lab, I continually glance back at my levitating object in wonder. I also glance up at the whiteboard and think that math is truly beautiful. I think my controller can best be explained by a story.

I was attempting to figure out a way to get the step response of the original system. It was not going terribly well. I could add a step, but I couldn't see anything at all through the noise from the system moving just a little at equilibrium (I'm currently thinking that a levitated object is not the best way to get the starting point that I then disturb with a step). Anyhow, I tried to stabilize the system by using a plastic rod to hold things together at the set point. That got rid of the high-amplitude, low-frequency noise, but high-frequency was still killing me. So I tried just adding a low-pass filter so that I might be able to see the step response better despite the noise. There was sadly too much noise. At this point I started poking around for a bit and one of the things I tried was changing the system to open-loop for a bit. I then took a look at the signal coming from the hall effect sensor. Importantly, I did this by touching the output of the hall-effect sensor to the place where my probe was at the time (after the low-pass filter). Then things moved and I was confused (I hadn't meant to change anything by probing the sensor's output). In my attempt to figure out what was going on, I inserted the sensor output into the row with the probe and (I guess out of habit?) went to set up the levitated object. It stayed. It stayed for a long, long time. And it had learned damping. It was incredible. I drew up the circuit on the board. I did math. It makes sense. It's absolutely incredible. I love math. I should still do that step response and system characterization thing, but I'll write up the math for why this works and include a block diagram first. So my design process was rather lacking, but I did figure everything out and fully understand it post facto. I was planning on doing the same thing with active elements, but this is actually more elegant. I'm glad I happened upon it.



By KCL:

$$I_{in} = I_R + I_C$$

By Ohm's Law:

$$I_R = \frac{V_{in} - V_{out}}{R}$$

$$I_C = \frac{V_{in} - 0}{\frac{1}{Cs}}$$

Using these:

$$I_{in} = \frac{V_{in} - V_{out}}{R} + \frac{V_{in} - 0}{\frac{1}{Cs}}$$

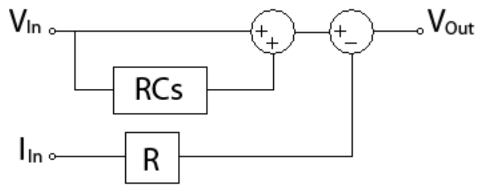
$$I_{in}\left(\frac{R}{Cs}\right) = V_{in}\left(\frac{1}{Cs} + R\right) - V_{out}\left(\frac{1}{Cs}\right)$$

$$V_{out} = Cs\left[V_{in}\left(\frac{1}{Cs} + R\right) - I_{in}\left(\frac{R}{Cs}\right)\right]$$

$$V_{out} = V_{in}\left(1 + RCs\right) - I_{in}R$$

$$V_{out} = V_{in}\,(1 + RCs) - I_{in}R$$

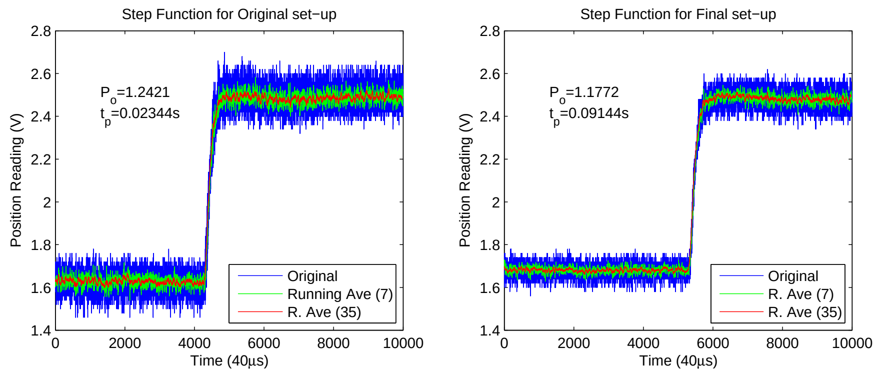
This gives us the following block diagram:



As we can see, this gives us an output that is effectively a PD controller. I_{in} is effectively constant so this only represents an offset. Given my value of R and the expected current sourcing from the sensor of $\approx 1mA$, this should give me an offset of $-10V$. That seems like a lot, but it's plausible that the derivative control could compete with it; in fact this would only require voltage movements with a time constant on the order of $100\mu s$. The most interesting part is that it manages to stay at a set position despite the fact that every other effect would originally seem to be much larger than the proportional control. Everything easily makes intuitive sense to within a constant, but I'll have to characterize the system in order to understand why this actually works. It'd be interesting to know how much R and C can change before the system stops being so well-behaved. I currently feel extraordinarily lucky - characterizing the system will show me if this is true or not. Let's try s'more characterizing.

System Characteristics

Step Functions



I actually expected to see a much, much bigger difference in the step functions. In particular, the step-function for the unstable system doesn't look as bad as I would've expected given the differences in stability. We can see, however, that the final system has a smaller overshoot and a slower rise - these are both characteristic of the damping that we were looking to implement with our compensator.

Transfer Functions

Let's start with our assumption that the world is second-order. This means we can get a characteristic transfer function using the following equations:

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$P_o = 1 + e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$$

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

Solving for the original system, we get:

$$\zeta = .5577$$

$$\omega_n = 201.5323$$

$$H(s) = \frac{40615.27}{s^2 + 224.7891s + 40615.27}$$

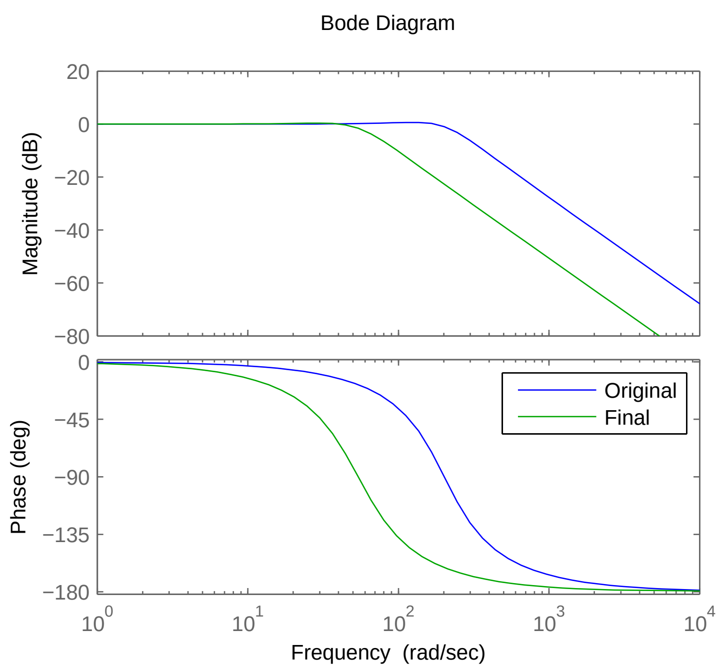
Solving for the final system, we get:

$$\zeta = .5960$$

$$\omega_n = 54.0515$$

$$H(s) = \frac{2921.56}{s^2 + 64.4294s + 2921.56}$$

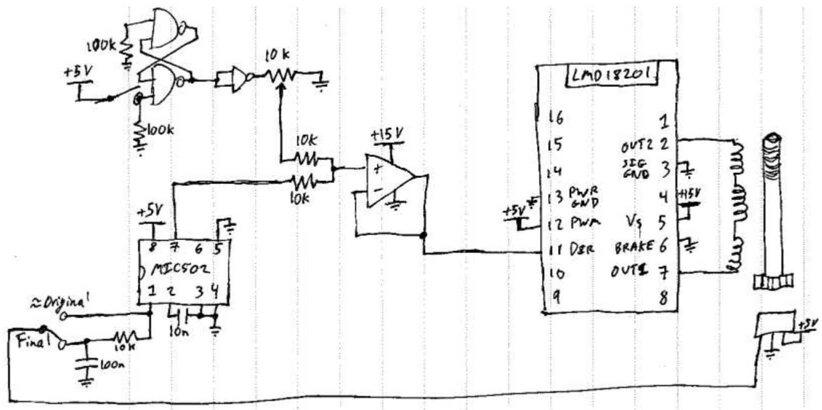
Bode Plots



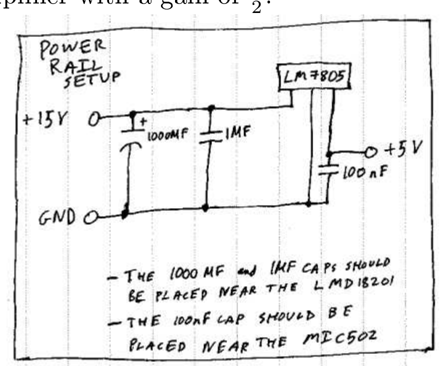
This is effectively what we should expect. The original system has more peaking and has a substantially higher cutoff frequency and peak frequency. Sweet. It's really cool to see that everything works. So cool.

At the end of the day, I think I was lucky, but not quite as lucky as I thought. I'd have to mess around with resistor and capacitor values in order to legitimately understand what range of values for R and C would stabilize the system. Effectively, I added a zero to the system which introduced damping; the R and C should set the position of the zero, but I have yet to develop intuition for how these are related.

Schematics



The nand gates are there to allow me to make a clean step. The cross-coupled nands make an sr latch that serves to debounce the switch and the other nand is simply used to get the step to have the appropriate orientation (low when disconnected, high when connected). The position labeled ≈original is called this because the op-amp effectively adds a gain of $\frac{1}{2}$ as compared to the original set-up. The resistor and capacitor at the final setting, implements a PD controller as shown in detail earlier. The op-amp is used as a summing amplifier with a gain of $\frac{1}{2}$.



References

Horowitz and Hill, Wikipedia (Operational_amplifier_applications), datasheets for all parts in kit, LMC6484 and SN74AC00.